On Theta Dependence of Glueballs from AdS/CFT

Gregory Gabadadze and Alberto Iglesias

Center for Cosmology and Particle Physics
Department of Physics, New York University, New York, NY, 10003, USA

Abstract

We study the theta dependence of the glueball spectrum in a strongly coupled cousin of large N gluodynamics defined via the AdS/CFT correspondence. By explicitly diagonalizing the 10d gravity equations in the presence of the RR 3-form and 1-form fluxes we found a mixing pattern for the lowest-spin lightest glueballs. The mixing between the scalar and pseudoscalar states is not suppressed, suggesting that the CP-odd effects persist in the large N theory. As a consequence, the lightest mass eigenstate ceases to be a parity eigenstate. We found the former as a linear combination of a scalar and pseudoscalar glueballs. On the other hand, the mass eigenvalues in a theory with and without the theta term remain equal in the large N limit.

1 Introduction and summary

QCD in the limit of a large number of colors, $N \to \infty$, is expected to contain a great deal of information on confinement [1]. In this limit gluons dominate the partition function and confinement should be easier to study in the theory without quarks. The subject of the present work is large N gluodynamics (i.e., QCD without quarks) with a nonzero theta term. Observationally, the theta parameter, if not zero, should be truly tiny, $\theta < 10^{-9}$, and, studies of the theta dependence in the large N theory may seem less motivated. However, this is not so because of the following arguments. Large N gluodynamics with the theta term reveals a rich ground state structure with multiple vacua separated by domain walls [2, 3, 4, 5]. This by itself is an extremely interesting manifestation of nonperturbative physics in a non-Abelian gauge theory. Moreover, certain remnants of the large N vacuum structure are expected to be present in real QCD too. Examples of this include the heavy nonperturbative states that make the domain walls [6], and new properties of axion and hadronic domain walls in a theory where the strong CP problem is solved by the axion mechanism (for recent summary see Ref. [7]).

The theta term enters the QCD action suppressed by one power of N compared to the gluon kinetic term. This may seem to indicate that no theta dependence should survive in the large N limit. For instance, a dilute instanton gas approximation would give rise to the theta dependence that is suppressed as $\exp(-N)$. However, there is a body of evidence (lattice and otherwise) suggesting that this is not so. The best example is the theta dependence of the vacuum energy [8, 2, 9, 3] which is of order $\mathcal{O}(N^0)$. This dependence can only be attributed to certain infrared effects that are not captured by a dilute instanton gas approximation. The related issue that has not been explored so far is the theta dependence of the spectrum of glueballs in this theory. Masses of glueballs are generated by noperturbative effects as well [10]. Similar effects could in general introduce the theta dependence in the spectrum even in the large N limit. In particular, we did not find convincing arguments to believe that the theta dependence would be completely washed out from the spectrum in the large N limit. How would one study these issues in more detail?

The theta term introduces a complex phase in the euclidean formulation of the partition function of a theory the action of which is otherwise real and positive semidefinite. Because of this lattice studies of these issues seem difficult (as an exception, see, *e.g.*, [11]). One way to address the question of the theta dependence of glueballs is to use the AdS/CFT correspondence [12, 13, 14] adapted to non-supersymmetric and non-conformal theories (for a review see, [15]). A well-known drawback of this approach is that it only allows to calculate gauge theory observables for a strongly coupled cousin of large N gluodynamics which in addition contains states that are not present in QCD. However, in the large N limit the effects of the additional states on QCD resonance physics should be minimized, and in the absence of other methods this seems to be a reasonable starting place.

In the main part of the work we study the theta dependence of the glueballs by looking at the IIA construction of N D4-branes with one compact worldvolume dimension [16] and a nonzero bulk RR 1-form flux. The latter gives rise to the theta dependence in the dual gauge theory [3]. In this setup we manage to diagonalize exactly the bulk gravity equations for the fluctuations of a graviton, dilaton and RR 1-form. The diagonalization of the gravity equations suggest the following remarkable pattern for the theta dependence of the gauge theory glueballs. The lowest scalar (0^{++}) and pseudoscalar (0^{-+}) glueballs mix. There are kinetic as well as mass mixing terms both of which are determined by θ. The system of glueballs can be diagonalized by shifting the field of the lightest spin-zero state by a field that is proportional to the heavier spin-zero glueball multiplied by θ. Interestingly enough, the above diagonalization can only be achieved by shifting the field of the lightest spin-zero state, which in gluodynamics is necessarily the 0^{++} glueball [17]. The heavier state remains intact. This property, as we will discuss in Section 4, is vital for the applicability of the method of the calculation. After the diagonalization, the lowest mass eigenstate ceases to be a parity eigenstate. The mixing between the parity eigenstates is a leading effect in large N (i.e., it is not suppressed by powers of N). However, the masses of the physical (diagonal) states in a theory with and without the theta term are the same. Hence, the only effect of the theta term in the leading large N limit is that the lowest mass spin-zero state becomes a mixed state of 0^{++} and 0^{-+} glueballs. The periodicity of the wavefunction with respect to $\theta \to \theta + 2\pi(\text{integer})$ is achieved by choosing appropriate branches of the theory in a way that also makes the vacuum energy periodic in θ [2, 3].

2 $D4$ Soliton with RR 1−form flux

In order to study the theta dependence of glueballs from the Gauge/Gravity correspondence we consider the dual supergravity description of pure $U(N)$ gauge field theory introduced in [16]. In this setup one starts with weakly coupled type IIA superstring theory in the presence of N D4-branes. The $D4$ *soliton* background solution relevant for this discussion is obtained by compactifying on a circle of radius M_{KK}^{-1} one of the 4 spacial directions of the 4-brane solution and by taking the near horizon limit. The effect of the compactification in the $D4$-branes worldvolume theory is that the fermions acquire masses of order M_{KK} at tree level due to the anti-periodic boundary conditions and so do the scalars due to loops. The worldvolume theory at energies below M_{KK} is then 4-dimensional nonsupersymmetric nonconformal $U(N)$ gauge theory. Thus we proceed to review the $D4$ soliton solution and how the effect of a theta term is incorporated.

The low energy effective action of type IIA superstrings in the Einstein frame and with vanishing Kalb-Ramond B field reduces to

$$I = \frac{1}{(2\pi)^7 l_s^8} \int \mathrm{d}^{10}x \sqrt{g} \left[g_s^{-2} \left(\mathcal{R} - \frac{1}{2}\partial_M \phi \partial^M \phi \right) - \frac{1}{4}\mathrm{e}^{3\phi/2}F_{(2)}^2 - \frac{1}{48}\mathrm{e}^{\phi/2}F_{(4)}^2 \right] , \quad (1)$$

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where l_s and g_s are the string length and coupling constant and $F_{(2)}$ and $F_{(4)}$ are the field strengths of the RR 1 and 3-forms respectively. From these, the equations of motion for the metric, dilaton and 1-form follow:

$$g_s^{-2}\left(\mathcal{R}_{MN} - \frac{1}{2}\partial_M\phi\partial_N\phi\right) = \frac{1}{2}e^{3\phi/2}\left(F_M{}^P F_{NP} - \frac{1}{16}g_{MN}F_2^2\right) -$$

$$- \frac{1}{48}e^{\phi/2}\left(4F_{MRST}F_N{}^{RST} - \frac{3}{8}g_{MN}F_{(4)}^2\right) , \quad (2)$$

$$g_s^{-2}\frac{1}{\sqrt{g}}\partial_M\left(\sqrt{g}g^{MN}\partial_N\phi\right) = \frac{3}{8}e^{3\phi/2}F_{(2)}^2 + \frac{1}{48}e^{\phi/2}F_{(4)}^2 , \quad (3)$$

$$\partial_M\left(\sqrt{g}e^{\phi/2}F^{MN}\right) = 0 . \quad (4)$$

The so called $D4$ *soliton* is given by the following Einstein frame metric, dilaton and constant 4-form:

$$\mathrm{d}s^2 = \left(\frac{R}{U}\right)^{3/8}\left[\left(\frac{U}{R}\right)^{3/2}\left(f(U)\mathrm{d}\tau^2 + \mathrm{d}x^\mu \mathrm{d}x_\mu\right) + \left(\frac{R}{U}\right)^{3/2}\frac{\mathrm{d}U^2}{f(U)} + R^{3/2}U^{1/2}\mathrm{d}\Omega_4^2\right](5)$$

$$e^\phi = \left(\frac{U}{R}\right)^{3/4} , \qquad\qquad F_{(4)} \sim g_s^{-1}\epsilon_4 ,$$

$$R^3 = \pi g_s N l_s^3 , \qquad\qquad f(U) = 1 - (U_{KK}/U)^3 , \quad (6)$$

where x^μ, $\mu = 1\ldots 4$ are the euclidean coordinates of the noncompact directions and τ denotes the coordinate along the compactified circle on the $D4$ worldvolume. $\mathrm{d}\Omega_4^2$ and ϵ_4 are the line element and volume form on a unit $4-$sphere and U is the coordinate on the radial direction transverse to the $D4$ branes. In order to avoid a conical singularity at $U = U_{KK}$, τ is identified with period $\Delta \equiv 2\pi M_{KK}^{-1} = 4\pi R^{3/2}/3U_{KK}^{1/2}$.

The supergravity description is valid in the regime of small curvatures (in string length units) and string coupling, namely

$$l_s^2\mathcal{R} \ll 1 , \qquad g_s e^\phi \ll 1 . \quad (7)$$

The latter implies a maximum value U_{max} of the radial coordinate that should therefore be much larger than the parameter U_{KK} [18]. This regime corresponds to the 't Hooft limit of the four dimensional theory $g_{YM} \to 0$, $N \to \infty$, $g_{YM}^2 N =$ fixed $\gg 1$, where the identification used $g_{YM}^2 \sim g_s l_s M_{KK}$ can be read off from (24) below.

The theta dependence of this background is obtained by turning on the RR $1-$form $C_{(1)}$ in the τ direction [3]:

$$C_\tau = -\frac{U_{KK}^3}{\Delta}\frac{\theta_c}{U^3} , \quad (8)$$

where $\theta_c = \theta + 2n\pi$ with integer n and the normalization has been chosen such that $\int \mathrm{d}\tau \mathrm{d}U F_{U\tau} = \theta_c$. Indeed, it is not difficult to see that (8) solves (4) if the

background geometry is not altered by the presence of this new field. This absence of backreaction on the geometry is due to the fact that in the large N limit the contribution from $F_{(2)}$ to (2) and (3) is subleading because the other terms contain extra powers of $g_s^{-1} \sim N$.

3 Supergravity modes

In order to find the glueball spectra, we proceed to study the equations of motion to linear order in the background of the previous section. In particular we are interested in the scalar modes that act as sources for the scalar and pseudoscalar operators of the boundary theory. For the case of vanishing theta this modes have been identified in [19] and [20]. In this section we will find the particular combination that diagonalizes the supergravity equations of motion when theta is turned on.

For the metric and dilaton perturbations we consider the following linearization

$$g_{MN} = g_{MN}^B + h_{MN} , \qquad \phi = \phi^B + \delta\phi , \tag{9}$$

where g_{MN}^B and ϕ^B stand for the background values and the diagonal components of the fluctuations are parametrized as follows:

$$
\begin{aligned}
h_{MM} &= g_{MN}^B v^N H(U) e^{ik_\mu x^\mu} , &(10)\\
\delta\phi &= \phi_0 e^{ik_\mu x^\mu} H(U) . &(11)
\end{aligned}
$$

Respecting the $SO(4)$ and $SO(5)$ symmetries of the $x^1 \ldots x^4$ and S^4 directions there are tree scalar fluctuations T, L and S (in the notation of [21]) that correspond to the dilaton, 4−sphere volume fluctuation and the exotic polarization [20] correspondingly. To simplify the expressions we choose $k^\mu = \delta^{\mu 4} k_4$ via an $SO(4)$ rotation. These are given by:

$$
\begin{aligned}
T: \quad v^N &= \frac{1}{4}(1, -\frac{5}{3}, -\frac{5}{3}, -\frac{5}{3}, 1, 1, 1, 1, 1, 1) , \quad \phi_0 = \frac{3}{2} , &(12)\\
L: \quad v^N &= (-1, -1, -1, -1, -1, -1, 1, 1, 1, 1) , \quad \phi_0 = -\frac{2}{3} , &(13)\\
S: \quad v^N &= \frac{1}{20}(-31, 9, 9, 9, \frac{-98 + 5U^3}{-2 + 5U^3}, \frac{-98 + 5U^3}{-2 + 5U^3}, 1, 1, 1, 1) , \quad \phi_0 = \frac{3}{10} . &(14)
\end{aligned}
$$

The exotic polarization S has also (in the gauge chosen) off diagonal components

$$h_{Ux_4} = h_{x_4 U} = -i\frac{k_4}{k^2} \frac{72U^{25/8}}{(5U^3 - 2)^2} H(U) e^{ik_\mu x^\mu} . \tag{15}$$

For each of these polarizations, the function $H \equiv T, L, S$ respectively satisfies:

$$
\begin{aligned}
U(U^3 - 1)T'' + (4U^3 - 1)T' - k^2 U T &= 0 , &(16)\\
U(U^3 - 1)L'' + (4U^3 - 1)L' - (k^2 U - 18U)L &= 0 , &(17)\\
U(U^3 - 1)S'' + (4U^3 - 1)S' - U\left(k^2 - \frac{108U}{(5U^3 - 2)^2}\right)S &= 0 , &(18)
\end{aligned}
$$

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where, for convenience, we have rescaled the coordinates to form dimensionless quantities, namely

$$\tilde{\tau} = \frac{U_{KK}^{1/2}}{R^{3/2}}\tau \ , \qquad \tilde{x}^\mu = \frac{U_{KK}^{1/2}}{R^{3/2}}x^\mu \ , \qquad \tilde{U} = \frac{U}{U_{KK}} \ , \tag{19}$$

and dropped the tildes.

Let us now turn our attention to the fluctuations of C_τ. We linearize the equation of motion for the RR 1-form (4) with fluctuations given by

$$C_\tau = C_\tau^B + \chi(U)\mathrm{e}^{ik_\mu x^\mu} \ , \tag{20}$$

where C_τ^B is the background value.

The key point is that the resulting equation can be decoupled from the metric and dilaton by choosing

$$\chi = \frac{3}{2}V + \frac{3\theta_c}{2\pi}f(U)(T+S) \ , \tag{21}$$

with V satisfying the following equation:

$$U(U^3-1)V'' + 4(U^3-1)V' - k^2UV = 0 \ , \tag{22}$$

the equation for fluctuations of C_τ in the absence of theta term. Thus, we were able to diagonalize the supergravity dual of $SU(N)$ gauge theory in the presence of a CP violating term. This has important consequences that we discuss in the following section.

4 Couplings to boundary theory

We infer the coupling of the supergravity modes to gauge invariant operators of the four dimensional field theory by considering the Born-Infeld action describing the low energy worldvolume excitations of the $D4$ branes. The operators of interest for us are $f_{\mu\nu}f^{\mu\nu}$ and $f_{\mu\nu}\tilde{f}^{\mu\nu}$ for Yang-Mills field-strength $f_{\mu\nu}$ and its dual $\tilde{f}^{\mu\nu}$. Neglecting worldvolume scalars it reads (in string frame):

$$I_{BI} = \mathcal{T}_4\mathrm{Tr}\int \mathrm{d}^5x \left(\mathrm{e}^{-\phi}\sqrt{\det\left(g_{mn} + 2\pi\alpha' f_{mn}\right)} + \frac{1}{8}i(2\pi\alpha')^2\epsilon^{mnrst}C_m f_{nr}f_{st}\right) \ , \tag{23}$$

$x^m = \tau, x^1 \ldots x^4$ and $\mathcal{T}_4 = (2\pi)^{-4}g_s^{-1}l_s^{-5}$. Upon compactification and expansion to second order the couplings to the gauge invariant operators are obtained [19]. We give the expression in Einstein frame:

$$I_{BI} = \frac{\Delta}{4}\mathcal{T}_4(2\pi\alpha')^2\mathrm{Tr}\int \mathrm{d}^4x \left(\mathrm{e}^{-3\phi/4}\sqrt{g_{\tau\tau}}\sqrt{\det g_{\mu\nu}}f^2 + \frac{1}{2}iC_\tau f\tilde{f}\right) \ . \tag{24}$$

By neglecting the metric fluctuations proportional to k^μ that give no contribution when contracted with the conserved energy momentum tensor of the four dimensional field theory we get the following result:

$$I_{BI} = I_{BI}^B + \int \mathrm{d}^4x \left(\psi \mathcal{O}_4 + \chi \tilde{\mathcal{O}}_4 \right) , \tag{25}$$

where I_{BI}^B accounts for the contribution from the background metric and dilaton fields and where we have defined the scalar and pseudoscalar glueball operators $\mathcal{O}_4$ and $\tilde{\mathcal{O}}_4$ and the corresponding couplings ψ and χ as follows:

$$\mathcal{O}_4 = \frac{\Delta\sqrt{f(U)}}{16\pi^2 g_s l_s} f_{\mu\nu} f^{\mu\nu} , \tag{26}$$

$$\tilde{\mathcal{O}}_4 = \frac{3i\Delta}{32\pi^2 g_s l_s} f_{\mu\nu} \tilde{f}^{\mu\nu} , \tag{27}$$

$$\psi = T + S , \tag{28}$$

$$\chi = \frac{3}{2} V + \frac{3\theta_c}{2\pi} f\psi . \tag{29}$$

Note that the S_4−volume scalar fluctuation L decouples.

The AdS/CFT prescription determines the generating functional for the 4-dimensional field theory in terms of the low energy effective string theory partition function for on-shell fields

$$\langle \mathrm{e}^{\int \mathrm{d}^4x \varphi_0 \mathcal{O}} \rangle = e^{-I_{SG}[\varphi]} , \tag{30}$$

in which the r.h.s. containing the supergravity action I_{SG} is used as an approximation to the string theory partition function and φ_0 are the boundary values of the on-shell normalizable supergravity modes φ acting as sources for the field theory operator $\mathcal{O}$. Therefore, to obtain the glueball spectrum we can consider the variation of the r.h.s. of (30) with respect to boundary sources to obtain two point functions. For example, for pseudoscalar glueballs we would consider the following variation

$$\frac{\delta}{\delta V_0(x^\mu)} \frac{\delta}{\delta V_0(y^\mu)} \mathrm{e}^{-I_{SG}[V,\psi]}|_{V_0=0,\psi_0=0} = \langle \tilde{\mathcal{O}}_4(x)\tilde{\mathcal{O}}_4(y) \rangle . \tag{31}$$

On the other hand, for scalar glueballs we find a mixing. Not only both scalars T and S in the combination ψ act as a source for $\mathcal{O}_4$ but also, when θ_c is nonvanishing, ψ sources $\tilde{\mathcal{O}}_4$. The corresponding two point function is obtained from the following variation:

$$\left(\frac{\delta}{\delta\psi_0(x)} - \frac{\theta_c}{\pi} \frac{\delta}{\delta V_0(x)} \right) \left(\frac{\delta}{\delta\psi_0(y)} - \frac{\theta_c}{\pi} \frac{\delta}{\delta V_0(y)} \right) \mathrm{e}^{-I_{SG}[V,\psi]}| = \langle \mathcal{O}_4(x)\mathcal{O}_4(y) \rangle , \tag{32}$$

since $f(U) \to 1$ at the boundary[1].

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[1]Eq. (32) makes the values $\theta = \pm\pi$ special. This indeed should be so since one expects spontaneous CP violation to take place for $\theta = \pm\pi$ [2, 3]. In our approach the form of Eq. (32) is an artifact of the normalizations that we choose to use for the bulk fields.

The above results can be summarized as follows. The field ψ sources a linear combination of the operators $\mathcal{O}_4$ and $\tilde{\mathcal{O}}_4$, while the field V sources only $\tilde{\mathcal{O}}_4$. Hence, the diagonalization of the bulk gravity equations in terms of ψ and V suggests the following mixing pattern for spin-zero glueballs: The lowest state is a linear superposition of a former scalar and pseudoscalar states while the heavier state is intact (it is just a pseudoscalar state). The mass of the pseudoscalar state, determined by Eq. (22), is the same as in the $\theta = 0$ theory (determined in Refs. [19, 21]). So is the mass of the mixed lightest state. Does this pattern have any special meaning? It seems it does. To see this consider the following two two-point correlation functions

$$C(x) \equiv \langle \mathcal{O}_4(x)\mathcal{O}_4(0) \rangle - \langle \mathcal{O}_4(0) \rangle^2,$$

and

$$\tilde{C}(x) \equiv \langle \tilde{\mathcal{O}}_4(x)\tilde{\mathcal{O}}_4(0) \rangle - \langle \tilde{\mathcal{O}}_4(0) \rangle^2.$$

These correlators can be saturated by the corresponding orthonormal set of physical intermediate states. Then, the leading behavior of $C(x)$ and $\tilde{C}(x)$ at large euclidean x^2 is determined by the corresponding lightest states. In a theory without the theta term those states are scalar and pseudoscalar glueballs respectively. However, once the theta term is switched on the scalars and pseudoscalars can mix. In general this mixing could be arbitrary. If so, then the former scalar state would also contribute to the expression for $\tilde{C}(x)$. However, because the scalar is lighter than the pseudoscalar (and after mixing it can only become even lighter), this would mean that $\tilde{C}(x)$ at large x^2 is completely dominated by the residue of the former scalar state. If this were true, we would not be able to determine the properties of the pseudoscalar state by calculating $\tilde{C}(x)$. Fortunately, this does not happen. The contribution of the physical lightest state (which is the former scalar glueball) is exactly canceled in $\tilde{C}(x)$, as suggested by the diagonalization of the gravity equations.

Acknowledgments

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